Design of 3-*dof* Spherical Robotic Mechanisms

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Abstract

This paper uses the kinematic mapping into the image space of spherical displacements to design a multi degree of freedom spherical closed chain, the so called 3-dof *spherical robotic mechanism, to guide a body through an arbitrary number of orientations. The design problem is to determine where to mount the robotic mechanism and where to hold the workpiece. A numerical example is given for* 6 *orientations.*

Keywords robotic mechanisms, spherical mechanisms, rigid body guidance, approximate motion synthesis,

1 Introduction

This paper presents the design of the 3-*dof* spherical robotic mechanism. A spherical robotic mechanism is a multi degree of freedom simple spherical closed kinematic chain. In the case of 3-*dof* spherical robotic mechanisms the closed chain consists of two $3R$ spherical open chains, or triads, which connect the workpiece, or floating link, to the ground. To maximize the workspace of the mechanism the two links in each of the $3R$ open chains are chosen to be 90(deg) in length.

For facilitating the kinematic synthesis of the 3-*dof* spherical robotic mechanism we view its $3R$ spherical open chains as variable crank length spherical RR dyads and employ well known dyadic synthesis techniques for rigid body guidance.

In Bodduluri 1991 the solution to four position rigid body guidance for the spatial $4C$ mechanism was presented and in Larochelle 1994 a design procedure for an arbitrary number of prescribed positions was demonstrated. Here we extend the works of Ravani and Roth 1983, Bodduluri 1990, and Larochelle 1994 to the dimensional synthesis of spherical robotic mechanisms for n position rigid body guidance, see Fig. 1. The first step of the design process is to define the design goal of the mechanism in terms of the desired positions of the moving body. The spherical robotic mechanism in this work possess three degrees of freedom and may be viewed as a spherical $4R$ mechanism with variable crank lengths. The design procedure allows for bounds on the crank lengths of the robotic mechanism. In the extreme case that the two crank lengths are held fixed we have a spherical $4R$ mechanism which in general can guide a body through only five positions, see Suh and Radcliffe 1978. Therefore, we utilize an optimization procedure first derived by Ravani and Roth 1983 by which we vary the synthesis variables so as to minimize the position error of the mechanism through an arbitrary number of positions.

The optimization algorithm employed involves writing the kinematic constraint equation of the variable crank length spherical RR dyad using the components of a quaternion. We view these equations as constraint manifolds in the image space of spherical displacements, see Bottema and Roth 1979 and McCarthy 1990. The result is an analytical representation of the workspace of the dyad which is parameterized by its dimensional synthesis variables. We then combine two variable crank length spherical RR dyads to form a $4R$ closed chain. The constraint manifold of the variable crank length $4R$ mechanism is simply the intersection of the constraint manifolds of its two RR subchains. This intersection provides an analytical representation of the workspace of the robotic mechanism in the image space of spherical displacements. The optimization goal is to determine the design variables such that all of the prescribed positions are either: (1) in the workspace, or, (2) the workspace comes as close as possible to all of the desired positions. The result of the design process is a 3-

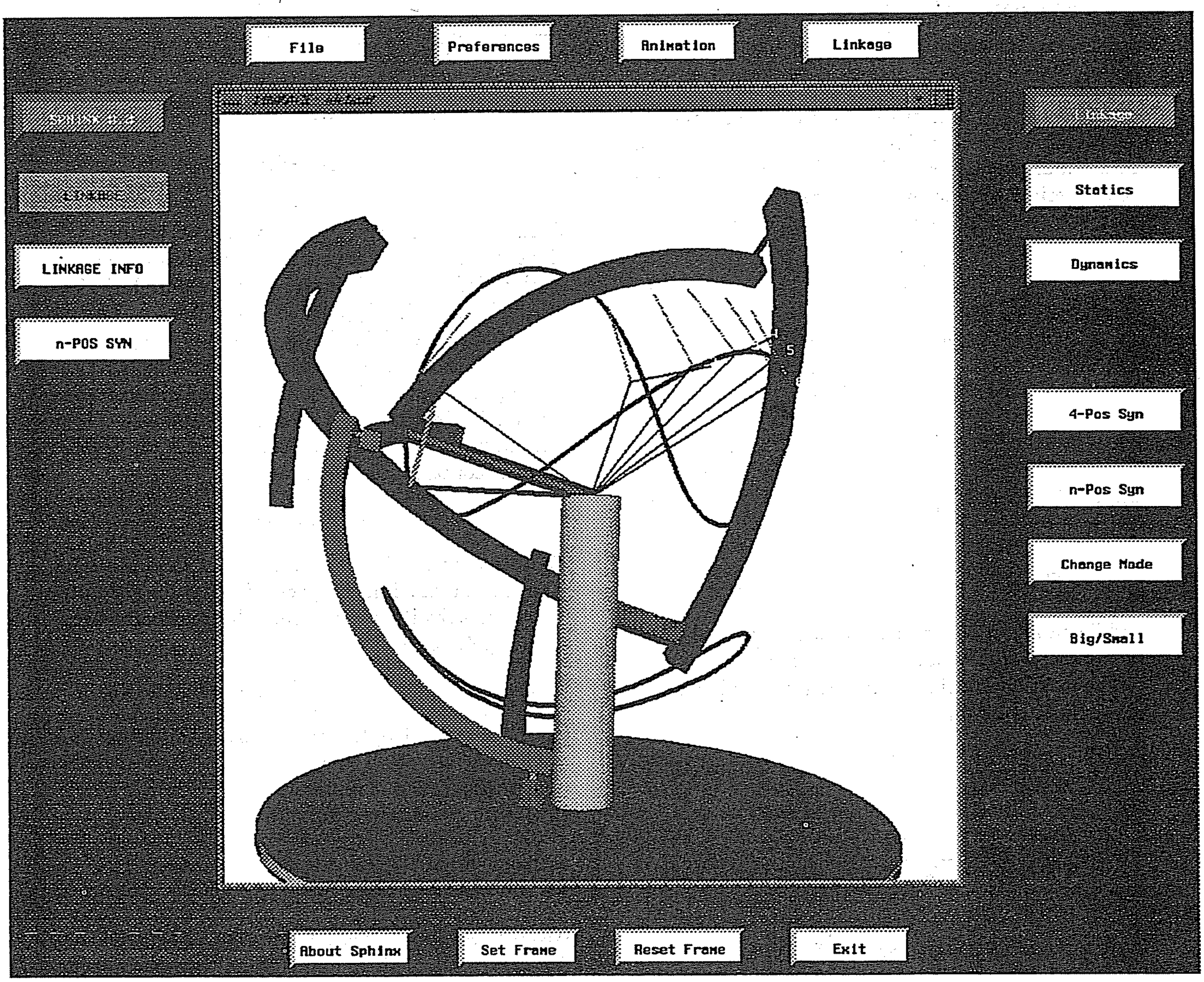

Figure 1: A 3-*dof* Spherical Robotic Mechanism

dof mechanism capable of providing general spherical motions. In what follows, we apply the optimization algorithm to the design of spherical mechanisms with six revolute joints, the 3-*dof* spherical robotic mechanism. A design case study for 6 desired orientations is presented.

2 Spherical Displacements

First, we review the use of quaternions for describing spherical displacements. A general spherical displacement may be described by a 3 × 3 orthonormal rotation matrix $[A]$. Using the rotation axis s and the rotation angle θ associated with $[A]$ we can represent the spherical displacement by the four dimensional vector, $\mathbf{q}$, which is written as, see McCarthy 1990,

$$\begin{aligned} q_1 &= s_x \sin\frac{\theta}{2} \\ q_2 &= s_y \sin\frac{\theta}{2} \\ q_3 &= s_z \sin\frac{\theta}{2} \\ q_4 &= \cos\frac{\theta}{2} \end{aligned} \quad (1)$$

We refer to $\mathbf{q}$ as a quaternion. The components of $\mathbf{q}$ satisfy,

$$G_s(\mathbf{q}) \;:\; q_1^2 + q_2^2 + q_3^2 + q_4^2 - 1 = 0 \quad (2)$$

and lie on a unit hypersphere which we denote as *the image space of spherical displacements.*

The rotation matrix, $[A]$ can be recovered from the quaternion, $\mathbf{q}$, as follows,

$$[A] = \quad (3)$$

$$\begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 - q_3q_4) & 2(q_1q_3 + q_2q_4) \\ 2(q_1q_2 + q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 - q_1q_4) \\ 2(q_1q_3 - q_2q_4) & 2(q_2q_3 + q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$

3 Constraint Manifolds

In this section we derive the algebraic constraint manifold of the spherical *RR* dyad. The continuous motion of the end link of an open chain maps into a constraint manifold in the image space. For closed chains the constraint manifold of the mechanism is the intersection of the constraint manifolds of its open subchains. The constraint manifolds are derived by using the geometric conditions that the joints of the dyad impose on the moving body. The vector equations for the geometric constraints are based upon the work of Suh and Radcliffe 1978, and Bodduluri 1990.

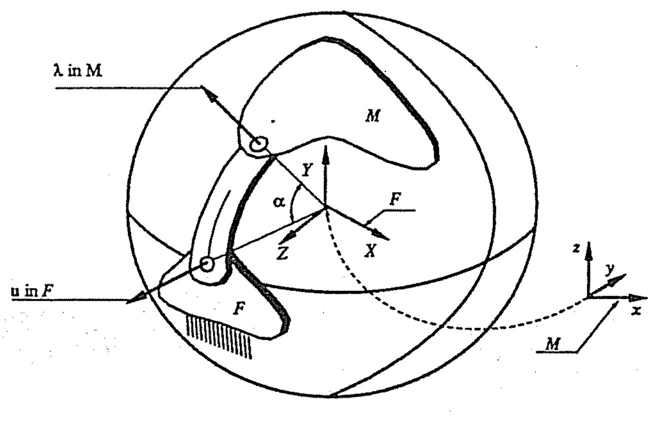

Figure 2: A Spherical *RR* Dyad

A spherical *RR* dyad is shown in Fig. 2. Let the axis of the fixed joint be specified by the vector $\mathbf{u}$ measured in the fixed reference frame F and let the moving axis be specified by $\boldsymbol{\lambda}$ measured in the moving frame M. Because the two axes are connected by a rigid link the angle between the two axes of the dyad remains constant and we have,

$$\mathbf{u} \cdot [A]\boldsymbol{\lambda} = \cos\alpha \quad (4)$$

To obtain an algebraic expression for the constraint manifold in the image space of spherical displacements we substitute Eq. 3 into the constraint equation, Eq. 4, and yield,

$$RR_{sph}(\mathbf{q},\mathbf{r}) : \quad \mathbf{u} \cdot [A(\mathbf{q})]\boldsymbol{\lambda} - \cos\alpha = 0 \quad (5)$$

Given $\mathbf{u}$, $\boldsymbol{\lambda}$, and α the constraint manifold of that dyad is the set of all image points, $\mathbf{q}$, that are solutions to Eq. 5 and it represents all possible locations of M with respect to F for the dyad.

4 Fitting Image Curves

We now describe the method presented by Ravani and Roth 1983 to perform dimensional synthesis using constraint manifolds. The first step is to formulate the constraint manifold of the mechanism, $CM(\mathbf{q},\mathbf{r})$.

$$CM(\mathbf{q},\mathbf{r}) : \begin{pmatrix} RR_{sph-a}(\mathbf{q},\mathbf{r}) \\ RR_{sph-b}(\mathbf{q},\mathbf{r}) \\ G_{sph}(\mathbf{q}) \end{pmatrix} = 0 \quad (6)$$

where $RR_{sph-a}(\mathbf{q},\mathbf{r})$ and $RR_{sph-b}(\mathbf{q},\mathbf{r})$ are from Eq. 5 written for each dyad of the mechanism, $G_s(\mathbf{q})$

is the quaternion constraint equation, Eq. 2, and $\mathbf{r}$ is the vector of dimensional synthesis variables for both dyads.

The goal is to determine the design variables $\mathbf{r}$ such that the constraint manifold passes through, or as close as possible to, the n desired points in the image space. Let $\mathbf{q}_d$ represent one desired point in the image space. We assume that $\mathbf{q}_d$ does not lie on the constraint manifold and write a Taylor series expansion of the constraint manifold about $\mathbf{q}_d$.

$$CM(\mathbf{q}_d, \mathbf{r}) + \frac{\partial CM(\mathbf{q}_d, \mathbf{r})}{\partial \mathbf{q}_d}(\mathbf{q} - \mathbf{q}_d) = 0 \qquad (7)$$

Let us now reformulate Eq. 7 as a system of linear equations,

$$[J]\mathbf{x} = \mathbf{b} \qquad (8)$$

where, $[J]$ is the matrix of partial derivatives of $CM(\mathbf{q}, \mathbf{r})$, $\mathbf{b} = -CM(\mathbf{q}_d, \mathbf{r})$, and $\mathbf{x} = \mathbf{q} - \mathbf{q}_d$. In general there will be infinite solutions to Eq. 8. Therefore, we seek the minimum norm solution of Eq. 8.

The minimum norm solution of Eq. 8 is found by use of the pseudo inverse of $[J]$,

$$[J]^{+} = [J]^T([J][J]^T)^{-1} \qquad (9)$$

which yields the minimum norm solution,

$$\mathbf{x}^* = [J]^{+}\mathbf{b} = \left\{[J]^T([J][J]^T)^{-1}\right\}\mathbf{b} \qquad (10)$$

From the definition of $\mathbf{x}$ we have,

$$\mathbf{q}^* = \mathbf{q}_d + \mathbf{x}^* \qquad (11)$$

where $\mathbf{q}^*$ approximates the point on the constraint manifold closest to $\mathbf{q}_d$. Moreover, we may use $\mathbf{q}^*$ to approximate the normal distance, $e(\mathbf{r})$, from $\mathbf{q}_d$ to the constraint manifold,

$$e(\mathbf{r})^2 = (\mathbf{q}^* - \mathbf{q}_d)^T(\mathbf{q}^* - \mathbf{q}_d) \qquad (12)$$

Finally, performing n position synthesis requires computing $e(\mathbf{r})$ for each desired position $\mathbf{q}_d$. The total error is then given by $E(\mathbf{r}) = \sum_{i=1}^{n} e_i^2(\mathbf{r})$ Thus, we have formulated the n position dimensional synthesis problem in the form of a nonlinear minimization problem with objective function $E(\mathbf{r})$. For further discussion of constraint manifold fitting see Larochelle 1994, Bodduluri 1990, and Ravani and Roth 1983.

5 Case Study

We present an example of the design of the 3-*dof* spherical robotic mechanism for 6 position rigid body guidance. The variable length cranks may be constructed by using a spherical isoceles triangle of equal sides of 90(deg) with the remaining side being the desired variable length crank. The desired crank length may be obtained by setting the interior angle between the 90(deg) links to the desired crank length. The 3-*dof* spherical robotic mechanism can also be viewed as two cooperating $3R$ spherical robots by considering each variable crank length dyad as a $3R$ spherical open chain and by viewing the coupler as the common workpiece to the robots. If the crank lengths are allowed to vary from 0(deg) to 90(deg) the system can reach any position on the sphere and each variable crank length dyad may be viewed as a $3R$ robot wrist.

The 6 desired positions are listed in Tbl. 1. The 12 element design vector $\mathbf{r}$ is,

$$\mathbf{r} = \begin{bmatrix} \mathbf{u}_a \\ \boldsymbol{\lambda}_a \\ \mathbf{u}_b \\ \boldsymbol{\lambda}_b \end{bmatrix} \qquad (13)$$

where $\mathbf{u}$ and $\boldsymbol{\lambda}$ are the fixed and moving axes of each open chain. Note that the variable crank lengths are not design variables to be sent to the optimization routine. That is because we allow α_a and α_b to vary during the motion of the robotic mechanism. Moreover, in this example we have constrained α_a and α_b such that,

$$75.0(\text{deg}) \leq \alpha_a \leq 85(\text{deg})$$
$$75.0(\text{deg}) \leq \alpha_b \leq 85(\text{deg})$$

We incorporate the variable crank lengths into the design procedure as follows.

1. Formulate an initial guess for the design vector from a spherical $4R$ mechanism which is a solution to three of the n desired positions.
2. Using the guess for the fixed and moving pivots solve for the two crank lengths such that the mechanism passes through the first position using the crank constraint equation, Eq. 4.
3. For each dyad, if the crank length is less than its lower bound then set it equal to its lower bound, if it is greater than its upper bound then set it equal to its upper bound.
4. Evaluate the error for this design to the first position.
5. Repeat steps $2-4$ for each of the n desired positions.
6. Send the design vector and the n position errors to the optimization routine.

Pos.	Long.	Lat.	Roll
1	−25.05	23.16	−40.09
2	30.00	20.00	15.00
3	45.00	25.00	20.00
4	60.00	30.00	25.00
5	75.00	30.00	20.00
6	90.00	30.00	0.00

Table 1: 6 Desired Positions

Pos.	DRIVING	DRIVEN	Error
1	84.99	84.96	$4.29E{-}14$
2	75.02	85.00	$6.10E{-}14$
3	78.22	84.46	$2.08E{-}14$
4	84.99	84.90	$2.11E{-}15$
5	83.65	82.33	$5.68E{-}14$
6	75.00	85.00	$4.97E{-}16$
			$\sum = 1.84E{-}13$

Table 2: Position Results

7. The result of the optimization routine is a better guess to the design vector. With the new design vector repeat steps $2-6$ until the algorithm has converged to a solution. If the total error is acceptable then the design is completed. If not, then select a new grouping of three of the n positions and repeat steps $1-7$.

The result of the the optimization procedure is the required crank lengths, found in Tbl. 2 with the resulting error at that position, and the initial guess and optimal design vectors for the linkage in Tbl. 3. The optimal design vector provides the location($\mathbf{u}_a$,$\mathbf{u}_b$) of the mechanism and the grasp points($\boldsymbol{\lambda}_a$,$\boldsymbol{\lambda}_b$) on the workpiece.

6 Conclusion

In this paper we have presented our development of an algorithm, originally proposed by Ravani and Roth 1983, for the dimensional synthesis of 3-*dof* spherical robotic mechanism to guide a body through an arbitrary number of orientations. The synthesis procedure utilizes an algebraic formulation for the constraint manifold of the spherical *RR* dyad to define the workspace of the robotic mechanism in the image space of spherical displacements. The result of the optimization is the location of the mechanism and the grasp points on the workpiece.

Design Vector **r**	
Initial Guess	Final Design
−0.72233	−0.86620
0.47945	0.20586
0.49837	0.45532
−0.80591	−0.61026
−0.16287	−0.59043
−0.56919	−0.52819
−0.95856	−0.90968
0.19023	0.31613
0.21206	−0.26934
−0.65721	−0.06470
−0.43955	−0.56649
−0.61227	−0.82153

Table 3: Optimization Results

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